

Prime Numbers 3

Any number can be expressed as a product of primes.

For example the number $12 = 2 \times 2 \times 3$

To express a number as a product of primes, try dividing it by 2 (the first prime) as many times as you can. Then try dividing it by the next prime (3) as many times as you can. Then try the next prime (5) as many times as you can and so on until you are left with 1.

eg Express 48 as a product of primes.

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

So $48 = 2 \times 2 \times 2 \times 2 \times 3$

eg Express 100 as a product of primes

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

So $100 = 2 \times 2 \times 5 \times 5$

Your teacher will show you some more examples.

Exercise.

Express each of the following as a product of prime factors.

1. 40
2. 58
3. 84
4. 95
5. 108
6. 312
7. 400
8. 600
9. 294
10. 1078
11. 1029
12. 1715
13. 3125
14. 550
15. 3185
16. 363
17. 384
18. 3375
19. 7875
20. 4096
21. Try this one ... 4093 (but don't try too hard!)

What is special about the number 4093?

22. Often $2^n - 1$ is prime.

For example, if $n=2$, $2^2 - 1 = 2 \times 2 - 1 = 3$

if $n=3$, $2^3 - 1 = 2 \times 2 \times 2 - 1 = 7$

if $n=4$, $2^4 - 1 = 2 \times 2 \times 2 \times 2 - 1 = 15$

if $n=5$, $2^5 - 1 = 2 \times 2 \times 2 \times 2 \times 2 - 1 = 31$

if $n=6$, $2^6 - 1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1 = 63$

if $n=7$, $2^7 - 1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1 = 127$

if $n=8$, $2^8 - 1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 - 1 = 255$

3 is prime

7 is prime

15 not prime

31 is prime

63 is not prime

127 is prime

255 is not prime

Often $2^n + 1$ is prime.

In the same way as above, test to see if it gives prime numbers for $n=1$, $n=2$,
 $n=8$.